In the late 80's almost periodic operators were one of the centers of attention of the mathematical community. Those operators serve in solid state physics as model of disordered systems, such as alloys, glasses and amorphous materials. One of the key results in the subject is the Kotani Theorem. It claims quite a specific spectral property of almost periodic Jacobi matrices: on the support of the absolutely continuous spectrum the operator should be reflectionless. Note that Sodin--Yuditskii under the assumption of regularity of the spectral set E with respect to the Lebesgue measure (the set should be homogeneous) proved the opposite statement: every Jacobi matrix, which is reflectionless on E, is almost periodic. A recent break-through result of Remling generated a new wave of interest in reflectionless operators. His theorem claims that each right limit of a Jacobi matrix is reflectionless on the support of the absolutely continuous spectrum of the initial operator. To indicate the importance of this result it is enough to say that in the simplest case when E is a single interval this theorem becomes the celebrated Rakhmanov theorem. In combination with our result it shows that if E is homo-geneous then each limit point is almost periodic. Let us point out that Remling's theorem does not require any assumption on E. Thus the following fundamental question arises: investigate properties of the right limits depending on the spectral set E beyond the homogeneous case. For instance Poltoratski and Remling recently proved that every reflectionless Jacobi matrix has purely absolutely continuous spectrum if E is weakly homogeneous. Our previous result stated that such matrices have absolutely continuous spectrum if the resolvent domain is of Widom type and the Direct Cauchy Theorem (DCT) holds on it. Now we constructed a Widom type domain such that a reflectionless matrix may have a singular continuous spectral component. We want to start our studies with the case when the set E consists of a family of intervals accumulating to the lower bound of E and with the question: find a geometric or analytic characterization of the set E such that the domain is of Widom type and the DCT holds on it. We are able to show that the DCT in such a case is intimately related with an L^1 extremal problem for entire functions with respect to the Lebesgue measure restricted to E. We give a precise setting of this problem on a modern level, which generalizes the classical one for polynomials and entire functions of a given exponential type. Similar to the polynomial case we plan to reduce the L^1 extremal problem to the question on orthogonalization of entire functions with respect to a special family of (reflectionless) weights. All these studies continue a very classical line of investigations in Analysis (Chebyshev problems, weighted polynomial approximations, spectral theory of canonical systems…) and, we believe, are of high independent interest.