

This project deals with the problem of resonance clustering appearing in nonlinear dispersive water wave systems governed by Euler equations. These wave systems are studied in the frame of wave turbulence theory, i.e. for the case nonlinearity is small (in a certain well-defined sense). Under this assumption, three wave turbulent regimes can be singled out: kinetic, with its description basing on statistical approach resulting in power energy spectra; discrete, defined by resonance clustering and covered by a set of dynamical systems; and mesoscopic, where both types of wave field evolution coexist. In various wave systems and under various experimental settings different regimes might occur. Moreover, including an additional physically relevant parameter may change observable regime as this is the case for capillary waves with and without rotation.

General importance of turbulent regimes with pronounced resonance clustering -- that is, discrete and mesoscopic regimes -- is due to the fact that resonances play major role in short-term forecast of wave field evolution in these systems (for instance, for ocean surface waves). A lot of studies of resonance clustering and its dynamics can be found in the literature but mostly for irrotational wave systems. However, rotation should be taken into account in a great amount of applications -- from technical equipment containing rotating tanks with fluids to various oceanic waves influenced by rotation of the Earth. This is our motivation for deeper study of resonance clustering in rotational wave systems.

The overall goal of this project is systematic study of resonance clustering in three water wave systems -- gravity waves, capillary waves and capillary-gravity waves -- with and without rotation which will be included in the form of constant non-zero vorticity. Results may be used in applications as a basic model for description of some known physical phenomena, e.g. freak waves.

The expected outcome of the project is (1) derivation of explicit formulae for resonance-generating vorticity for gravity, capillary and gravity-capillary waves; (2) construction of resonance clustering for exact and approximate magnitudes of vorticity; (3) derivation of the general form of coupling coefficient for each of these systems and investigating the conditions of effective resonant interactions, i.e. for the case when the coupling coefficient is not identically equal to zero; (4) studying the dependence of the resonance clustering (4a) on the wave system type, and (4b) on its space dimensions; (5) development of a discrete analog of energy cascades in mesoscopic regime and comparing our theoretical findings with results of laboratory experiments.

A thorough check of consistence of our theoretical results with experimental data will be conducted in close cooperation with the group of Prof. M. Shats (Australian National University, Canberra, Australia) possessing adequate experimental facilities as well as long-year practice in laboratory study of water waves and a vast amount of data thereto.