Complex Analysis, Spectral Theory and Approximation meet in Linz

Johannes Kepler University Linz, July 3 - 8, 2022

Program & Abstracts

Monday 04.07.2022

09'00 - 09'30 `Registration (Uni Center Loft B, 2nd floor)`

`- Uni Center Loft C, 2nd floor-`

09'30 - 10'30	*Mikhail Sodin:* From Kharkiv to Linz with stops in Melitopol, E. Lansing and Ramat Gan

10'30 - 11'00 `Coffee Break (Uni Center Loft D, 2nd floor)`

`- Uni Center Loft C, 2nd floor -`

11'00 - 12'00	*Barry Simon:* A Tale of Three Coauthors: Comparison of Ising Models

12'00 - 14'00 `Lunch Break`

`- Uni Center Loft C, 2nd floor -`

14'00 - 15'00	*Stanislas Kupin:* On spectral properties of compact Toeplitz operators on Bergman space with logarithmically decaying symbol and applications to banded matrices

15'00 - 15'30 `Coffee Break (Uni Center Loft D, 2nd floor)`

`- Uni Center Loft C, 2nd floor -`

15'30 - 16'30	*Sergey Tikhonov:* Inequalities for Fourier transforms

16'30 - 17'30 `Poster Session (Uni Center Loft B, 2nd floor)`

`- Uni Center Loft C, 2nd floor -`

17'30 - 18'30	*Alexandre Eremenko:* Second order linear differential equations with a basis of solutions having only real zeros

Tuesday 05.07.2022

09'00 - 09'30 `Registration (Uni Center Loft B, 2nd floor)`

`- Uni Center Loft C, 2nd floor-`

09'30 - 10'30	*Sergey Denisov:* Spectral theory of Jacobi matrices on trees whose coefficients are generated by multiple orthogonality

10'30 - 11'00 `Coffee Break (Uni Center Loft D, 2nd floor)`

`- Uni Center Loft C, 2nd floor -`

11'00 - 12'00	*Roman Bessonov:* Szegő measures and vibration of Krein strings

12'00 - 14'00 `Lunch Break`

`- Uni Center Loft C, 2nd floor -`

14'00 - 15'00	*Gerald Teschl:* Long time asymptotics of Toda shock waves
15'10 - 15'40	*Iryna Karpenko:* The modified Camassa-Holm equation on a step-like background
15'50 - 16'20	*Giorgio Young:* Orthogonal rational functions with real poles, root asymptotics, and GMP matrices

16'20 - 17'00 `Coffee Break (Uni Center Loft D, 2nd floor)`

`- Uni Center Loft C, 2nd floor -`

17'00 - 18'00	*Fritz Gesztesy:* Sturm-Liouville M-Functions in Terms of Green's Functions

Wednesday 06.07.2022

09'00 - 09'30 `Registration (Uni Center Loft B, 2nd floor)`

`- Uni Center Loft C, 2nd floor-`

09'30 - 10'30	*Milivoje Lukic:* An approach to universality using Weyl m-functions

10'30 - 11'00 `Coffee Break (Uni Center Loft D, 2nd floor)`

`- Uni Center Loft C, 2nd floor -`

11'00 - 11'30	*Gökalp Alpan:* Extremal polynomials on a Jordan arc and compact subsets of the real line
11'40 - 12'10	*Fei Xu:* Infinite Dimensional ODEs with Discrete Convolution of Higher Dimension and Combinatorial Analysis

12'10 - 14'00 `Lunch Break`

`- Uni Center Loft C, 2nd floor -`

14'00 - 15'00	*Jacob Christiansen:* Chebyshev polynomials in the complex plane

15'00 - 15'30 `Coffee Break (Uni Center Loft D, 2nd floor)`

`- Uni Center Loft C, 2nd floor -`

15'30 - 16'30	*Jake Fillman:* Spectra of periodic and limit-periodic operators

16'30 - 17'00 `Coffee Break (Uni Center Loft D, 2nd floor)`

`- Uni Center Loft C, 2nd floor -`

17'00 - 18'00	*David Damanik:* Fun and games with gap labelling

Thursday 07.07.2022

09'00 - 09'30 `Registration (Uni Center Loft B, 2nd floor)`

`- Uni Center Loft C, 2nd floor -`

09'30 - 10'30	*Alexander Aptekarev:* L^p bounds for orthogonal polynomials and applications

10'30 - 11'00 `Coffee Break (Uni Center Loft D, 2nd floor)`

`- Uni Center Loft C, 2nd floor -`

11'00 - 12'00	*Iryna Egorova:* Long-time asymptotics of steplike solutions of KdV equation
12'10 - 12'40	*Aleksei Kulikov:* Contractions between Hardy and Bergman spaces

13'00 - 19'00 `Excursion`

19'00 – `Conference Dinner`

Friday 08.07.2022

09'00 - 09'30 `Registration (Uni Center Loft B, 2nd floor)`

`- Uni Center Loft C, 2nd floor-`

09'30 - 10'30	*Alexander Kheifets:* Hardy Spaces of Fuchsian Groups for Akhiezer-Levin points

10'30 - 11'00 `Coffee Break (Uni Center Loft D, 2nd floor)`

`- Uni Center Loft C, 2nd floor -`

11'00 - 12'00	*Aleksey Kostenko:* Generalized indefinite strings

12'00 - 14'00 `Lunch Break`

`- Uni Center Loft C, 2nd floor -`

14'00 - 15'00	*Harald Woracek:* Spectral theory of canonical systems: discreteness of spectrum and density of eigenvalues
15'10 - 15'40	*Jakob Reiffenstein:* A trace class criterion for canonical systems

15'40 - 16'00 `Coffee Break (Uni Center Loft D, 2nd floor)`

`- Uni Center Loft C, 2nd floor -`

16'00 - 17'00	*Alexander Volberg:* Dyadic rectangles or do we know the Carleson embedding on bi-disc?

CASAT 2022 – Abstracts

Extremal polynomials on a Jordan arc and compact subsets of the real line

Alpan Gökalp `Wednesday 11'00 (Loft C)`

I briefly describe Widom's theory of extremal polynomials on a system of smooth Jordan curves and arcs. Then I discuss some recent improvements on the upper estimate for weighted Chebyshev polynomials given in Widom's seminal paper (1969). I also survey optimal lower and upper bounds for the norms of weighted Chebyshev polynomials and orthogonal polynomials on the real line.

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L^p bounds for orthogonal polynomials and applications

Aptekarev Alexander `Thursday 09'30 (Loft C)`

The classical Steklov problem deals with bounds of the Tchebyshëv norm $\|p_n\|^\infty(\Delta)$ for the polynomials $p_n(x)$, orthonormal with respect to the strictly positive wieght function $w \in L^1(\Delta)\bigcap S_\delta,\ S_\delta := \{w : w(x) \geq \delta > 0, x \in \Delta\}$. Modern applications (in particular, to the information entropy of quantum systems) motivate us to consider also the estimates of L^p norms: $\|p_n\|^p_w(\Delta)$ for the Steklov weight functions $w \in X(\Delta)\bigcap S_\delta$ from the various classes $X := L^\infty$, S — (the Szego class), BMO, A_p — (the Muckenhoupt class).

Our talk is based on the joint papaer with Sergey Denisov and Michel Alexis [1]. Thus, we focus on $\|p_n\|^p_w,\ p > 2$, for $w \in A_2\bigcap S_\delta$.

References:

1. M. Alexis, A. Aptekarev and S. Denisov, *Continuity of Weighted Operators, Muckenhoupt Ap Weights, and Steklov Problem for Orthogonal Polynomials* // International Mathematics Research Notices, Vol. 2022, No. 8, pp. 5935–5972.

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Szegő measures and vibration of Krein strings

Bessonov Roman `Tuesday 11'00 (Loft C)`

We give a dynamical characterization of Szego measures on the real line. Szego condition for a measure $\mu = w\,dx + \mu_s$

$$\int_{\mathbb{R}} \frac{\log w(x)}{1+x^2}\,dx > -\infty$$

is proved to be equivalent to stable propagation of waves on an associated Krein string. Related results in scattering theory of Dirac operators and canonical systems will be also discussed. Joint work with Sergey Denissov.

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Chebyshev polynomials in the complex plane

Christiansen Jacob `Wednesday 14'00 (Loft C)`

In the talk, I'll discuss the theory for Chebyshev polynomials in the complex plane. These are the monic polynomials, T_n, which minimize the sup-norm on a given compact infinite set $\mathsf{E} \subset \mathbb{C}$.

It is natural to ask how the norms $\|T_n\|_{\mathsf{E}}$ relate to the theoretical lower bound $\mathrm{Cap}(\mathsf{E})^n$ and how the zeros of T_n distribute as $n \to \infty$. I'll address these and related questions for various classes of sets E (e.g., lemniscates and polynomial preimages).

Much of what I'll say is motivated by graphs and plots produced using a refined version of the complex Remez algorithm and is largely based on joint work in progress with O. Rubin (Lund University).

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Fun and games with gap labelling

Damanik David `Wednesday 17'00 (Loft C)`

We will give an introduction to Johnson's gap labelling based on the Schwartzman homomorphism and describe several recent applications of this theory, including a positive answer to a question of Bellissard about the structure of the almost sure spectra of random Schrödinger operators.

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Spectral theory of Jacobi matrices on trees whose coefficients are generated by multiple orthogonality

Denisov Sergey `Tuesday 09'30 (Loft C)`

The connection between the polynomials orthogonal on the real line and Jacobi matrices is well-known. I will explain how self-adjoint Jacobi matrices on trees appear naturally in the theory of multiple orthogonal polynomials. For two classical systems, e.g., Angelesco and Nikishin systems, the properties of the corresponding Jacobi matrices will be discussed. This work is based on a series of papers with A. Aptekarev and M. Yattselev.

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Long-time asymptotics of steplike solutions of KdV equation

Egorova Iryna `Thursday 11'00 (Loft C)`

Asymptotics for the Korteweg - de Vries equation with steplike initial data on the constant backgrounds have long been well studied at the physical level of rigor.

This talk will present some recent mathematically rigorous results that refine and justify the above-mentioned asymptotics.

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Second order linear differential equations with a basis of solutions having only real zeros

Eremenko Alexandre `Monday 17'30 (Loft C)`

We study differential equations $w'' + Aw = 0$, where A is an entire function. When A is a polynomial, two linearly independent solutions having only real zeros are possible only when A is constant. However there are transcendental entire functions A with this property, and we describe possible orders and asymptotic behavior of such functions A of finite order.

Based on the joint work with Walter Bergweiler and Lasse Rempe.

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Spectra of periodic and limit-periodic operators

Fillman Jake `Wednesday 15'30 (Loft C)`

We will discuss spectral properties of periodic and limit-periodic operators, with an emphasis on results about the size of the spectrum in topological, measure-theoretic, and fractal senses. The main theme will focus on a series of results showing that the spectra of limit-periodic operators may be Cantor sets that are very thin in all three senses: they are Cantor sets of zero measure which also have zero Hausdorff dimension. We will discuss some recent work that leverages inverse spectral results and noncommutation arguments to give robust proofs of thin spectra for a wide variety of models.

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Sturm-Liouville M-Functions in Terms of Green's Functions

Gesztesy Fritz `Tuesday 17'00 (Loft C)`

We discuss a reformulation of Weyl–Titchmarsh theory for (three-coefficient) regular and singular Sturm–Liouville operators for separated and coupled self-adjoint boundary conditions (if any) in terms of the (diagonal) Green's function and some of its first quasi-derivatives. In particular, this Green's function approach unifies the treatment of separated and coupled boundary conditions (the treatment of the latter appears to be new in this context).

This is based on joint work with Roger Nichols (Univ. of Tennessee at Chattanooga).

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The modified Camassa-Holm equation on a step-like background

Karpenko Iryna `Tuesday 15'10 (Loft C)`

We consider the initial value problem for the modified Camassa–Holm (mCH) equation:

$$m_t + \left((u^2 - u_x^2)m\right)_x = 0, \quad m := u - u_{xx}, \quad t > 0, \ -\infty < x < +\infty, \tag{1a}$$

$$u(x,0) = u_0(x), \qquad -\infty < x < +\infty, \tag{1b}$$

assuming that

$$u_0(x) \to \begin{cases} A_1 \text{ as } x \to -\infty \\ A_2 \text{ as } x \to \infty \end{cases},$$

where A_1 and A_2 are some different non-zero constants, and that the solution $u(x,t)$ preserves this behavior for all fixed $t > 0$.

The mCH equation was introduced as a new integrable system by Fuchssteiner, Olver and Rosenau, and arises as a model equation in the theory of nonlinear water waves. It is integrable in the sense that it has a Lax pair representation, which allows, in principal, developing the inverse scattering method for studying the properties of solutions of Cauchy problems in various classes of functions.

We develop the inverse scattering transform method in the form of Riemann–Hilbert (RH) problem for the Cauchy problem (1) assuming that $m(x,0) = u_0(x) - u_{0xx}(x) > 0$ for all x (then it can be shown that $m(x,t) > 0$ for all t). Particularly, we present detailed properties of spectral functions associated with the initial data for the Cauchy problem for the mCH equation and obtain a representation for the solution of this problem in terms of the solution of an associated RH problem.

The talk is based on a joint work with D. Shepelsky and G. Teschl.

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Hardy Spaces of Fuchsian Groups for Akhiezer-Levin points

Kheifets Alexander `Friday 09'30 (Loft C)`

Let L^2 be the space of functions on the unit circle $\mathbb{T}$ with

$$\int_{\mathbb{T}} |f(t)|^2 \mu(dt) < \infty,$$

where μ is the Lebesgue measure. Let $\mathcal{L}^2_{t_0}$ be the space of functions with

$$\int_{\mathbb{T}} |f(t)|^2 \frac{\mu(dt)}{|t-t_0|^2} < \infty, \quad |t_0| = 1.$$

Let H^2 and $\mathcal{H}^2_{t_0}$ be the corresponding Hardy spaces. Let Γ be a Fuchsian group. Let α be a character of Γ. $H^2(\alpha)$ and $\mathcal{H}^2_{t_0}(\alpha)$ are subspaces that consist of α-automorphic functions. These spaces are convenient, in particular, as models for almost periodic finite difference and differential operators, respectively.

Celebrated results of Widom and Pommerenke give necessary and sufficient conditions on Γ for the space $H^2(\alpha)$ to be nontrivial for every character α of Γ. This characterization was given in terms of the Green function of the group Γ. Our result is an analogue of their result for $\mathcal{H}^2_{t_0}(\alpha)$. In our characterization the Green function is replaced with the Martin function of t_0. In addition to the Widom-Pommerenke type condition, there is also a condition that the Martin function must be a pure point one.

Joint work with P. Yuditskii.

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Generalized indefinite strings

Kostenko Aleksey `Friday 11'00 (Loft C)`

The aim of this talk is to overview our joint work with J. Eckhardt (Loughborough University, UK) on generalized indefinite strings, the object first appeared in the work of M.G. Krein and H. Langer on the indefinite moment problem. The importance of a generalized indefinite string stems, on the one hand, from the fact that it provides yet another canonical model of a self-adjoint operator with simple spectrum, and on the other hand, its connection with the Camassa-Holm equation, for which it serves as an isospectral problem/Lax operator.

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Contractions between Hardy and Bergman spaces

Kulikov Aleksei `Thursday 12'10 (Loft C)`

Consider spaces of analytic functions f in the unit disc $\mathbb{D} = \{z : |z| < 1\}$ such that $|f(z)|^p$ is integrable with respect to the weight $\frac{\alpha+1}{\pi}(1-|z|^2)^\alpha$ for some $0 < \pi < \infty$, $\alpha > -1$. It was essentially known since the work of Hardy and Littlewood which of these spaces are contained in which. Recently, however, it was asked whether these embeddings are actually contractions, which turns out to be related to some open questions in mathematical physics and analysis of functions of infinitely many variables.

In this talk we will discuss our recent work in which we confirm this conjecture as well as more generally and the maximum value for monotone and convex functionals on Hardy and Bergman spaces, respectively. The proof is based on a combination of tools from complex analysis and hyperbolic geometry.

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On spectral properties of compact Toeplitz operators on Bergman space with logarithmically decaying symbol and applications to banded matrices

Kupin Stanislas Monday 14'00 (Loft C)

Let $L^2(\mathbb{D})$ be the space of measurable square-summable functions on the unit disk. Let $L^2_a(\mathbb{D})$ be the Bergman space, *i.e.*, the (closed) subspace of analytic functions in $L^2(\mathbb{D})$. P_+ stays for the orthogonal projection going from $L^2(\mathbb{D})$ to $L^2_a(\mathbb{D})$. For a function $\phi \in L^\infty(\mathbb{D})$, the Toeplitz operator $T_\phi : L^2_a(\mathbb{D}) \to L^2_a(\mathbb{D})$ is defined as

$$T_\phi f = P_+ \phi f, \quad f \in L^2_a(\mathbb{D}).$$

In this talk, we present a result on spectral asymptotics for singular (or eigen-) values of compact Toeplitz operators with logarithmically decaying symbols, that is

$$\phi(z) = \phi_1(e^{i\theta}) \, (1 + \log(1/(1-r)))^{-\gamma}, \quad \gamma > 0,$$

where $z = re^{i\theta}$ and ϕ_1 is a continuous (or piece-wise continuous) function on the unit circle. It is then applied to the spectral analysis of banded (including Jacobi) matrices.

This is a joint work with S. Naboko[†] (St. Petersburg, Russia), M. Koita and B. Touré (Ségou, Mali).

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An approach to universality using Weyl m-functions

Lukic Milivoje Wednesday 09'30 (Loft C)

I will describe an approach to universality limits for orthogonal polynomials on the real line which is completely local and uses only the boundary behavior of the Weyl m-function at the point. We show that bulk universality of the Christoffel–Darboux kernel holds for any point where the imaginary part of the m-function has a positive finite nontangential limit. This approach is based on studying a matrix version of the Christoffel–Darboux kernel and the realization that bulk universality for this kernel at a point is equivalent to the fact that the corresponding m-function has normal limits at the same point. Our approach automatically applies to other self-adjoint systems with 2×2 transfer matrices such as continuum Schrödinger and Dirac operators. We also obtain analogous results for orthogonal polynomials on the unit circle. This is joint work with Benjamin Eichinger and Brian Simanek.

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A trace class criterion for canonical systems

Reiffenstein Jakob Friday 15'10 (Loft C)

This talk is a follow-up to the talk of H. Woracek. Consider a two-dimensional canonical system $y'(t) = zJH(t)y(t)$ on the half-line, with positive semi-definite Hamiltonian H. Let A_H be the differential operator associated to the system, with boundary condition $y_1(0) = 0$. We give an explicit characterization in terms of H for the resolvents of A_H to belong to a Schatten-von Neumann class $\mathfrak{S}_p$, where $p \in (0, 2)$. Most notably, setting $p = 1$ yields a criterion for trace class.

Our results rests on the observation that we can view the fundamental solution as an aggregate of Weyl coefficients of cut-offs of H. Recent results on the high-energy behaviour of Weyl coefficients allow us to roughly determine the growth of the fundamental solution. Classical Abelian-Tauberian theorems then lead to equivalent growth and summability conditions for the eigenvalues.

The talk is based on joint work with Matthias Langer, Raphael Pruckner and Harald Woracek.

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A Tale of Three Coauthors: Comparison of Ising Models

Simon Barry Modnay 11'00 (Loft C)

On Friday, Jan 14, I had a draft of a single author paper intended for the Lieb Festschrift. Six days later, the paper had three coauthors. This talk will explain the interesting story, expose some underlying machinery and sketch the proof of a lovely inequality on certain finite sums.

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From Kharkiv to Linz with stops in Melitopol, E. Lansing and Ramat Gan

Sodin Mikhail Monday 09'30 (Loft C)

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Long time asymptotics of Toda shock waves

Teschl Gerald Tuesday 14'00 (Loft C)

This talk is concerned with the long time asymptotics of the Toda lattice with steplike initial conditions leading to shock waves. It turns out that in this shock wave consists of a middle region which has the form of a two-band solution plus two transitional regions where the wave can be asymptotically described as a modulated elliptic solution. This result will be rigorously established using the nonlinear steepest descent method for oscillatory Riemann—Hilbert problems.

Based on joint work with Iryna Egorova, Johanna Michor and Anton Prymimak

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Inequalities for Fourier transforms

Tikhonov Sergey Monday 15'30 (Loft C)

We discuss classical Hausdorff–Young and Hardy–Littlewood inequalities for Fourier coefficients/transforms as well as their possible extensions for the whole range of the integrability parameter.

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Dyadic rectangles or do we know the Carleson embedding on bi-disc?

Volberg Alexander Friday 16'00 (Loft C)

Carleson embedding theorem often serves as a first building block for interpolation in complex domain, for the theory of Hankel operators and in PDE. The other name is paraproducts estimates. The embedding of certain spaces of holomorphic functions on n-polydisc can be reduced (without loss of information) to the boundedness of weighted multi-parameter dyadic Carleson embedding. We find the necessary and sufficient condition for this Carleson embedding in n-parameter case, when n is 2, or 3. The main tool is the harmonic analysis on graphs with cycles. The answer is quite unexpected and seemingly (only seemingly) goes against Carleson quilts counterexample of 1974. The main tool is an unexpected combinatorial properties of positive measures on cube in dimensions 2, 3. I will present results obtained jointly by Arcozzi, Holmes, Mozolyako, Psaromiligkos, Zorin-Kranich and myself. But spaces of holomorphic functions in bi-disc mentioned above do not include the classical Hardy space on bi-disc. Why we do not know the embedding theorem for Hardy space on bi-disc? There is a deep reason for that, which will be explained.

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Spectral theory of canonical systems: discreteness of spectrum and density of eigenvalues

Woracek Harald Friday 14'00 (Loft C)

We consider the spectral theory of the differential operator A_H associated with a two-dimensional canonical system $y'(t) = zJH(t)y(t)$ on the half-line. The Hamiltonian H of the system is assumed to positive semidefinite a.e. and trace-normalised. Two natural questions are:

1. Is the spectrum of A_H discrete ?
2. If $\sigma(A_H)$ is discrete, how "dense" are the eigenvalues λ_n ?

Of course, here one has to make precise what is meant by "density" of a sequence: we use as a measure for density for example the convergence exponent of the sequence $(1/\lambda_n)$, or its membership in certain Lorentz ideals. The first question has an astonishingly simple and explicit answer. The answer to the second question is technically more involved, still explicit in terms of H and applicable in practice.

One crucial ingredient is that membership in operator ideals of resolvents of A_H are independent of the off-diagonal entries of H when the operator ideal is sufficiently large. And by this we mean that the ideal is slightly larger than trace-class; for example every Schatten-von Neumann class $\mathfrak{S}_p$ with $p > 1$ will work.

When coming close to trace-class, or going beyond $\mathfrak{S}_1$ to even more sparse sequences of eigenvalues, the method breaks down completely (for intrinsic and natural reasons). To investigate the situation that eigenvalues are sparse, very different methods are needed, and those will be presented in the follow-up talk by J.Reiffenstein.

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Infinite Dimensional ODEs with Discrete Convolution of Higher Dimension and Combinatorial Analysis

Xu Fei Wednesday 11'40 (Loft C)

In this talk, we will introduce the existence and uniqueness of spatially quasi-periodic solutions to the generalized KdV equation on the real line. In the Fourier space, the quasi-periodic Cauchy problem is equivalent to the infinite dimensional ODEs with discrete convolution of higher dimension. By applying a combinatorial analysis method, we obtain the local result. This is based on the joint work with David Damanik (Rice University) and Yong Li (Jilin University).

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Orthogonal rational functions with real poles, root asymptotics, and GMP matrices

Young Giorgio Tuesday 15'50 (Loft C)

There is a vast theory of the asymptotic behavior of orthogonal polynomials with respect to a measure on $\mathbb{R}$ and its applications to Jacobi matrices. That theory has an obvious affine invariance and a very special role for ∞. In this talk, I will discuss work extending aspects of this theory to the setting of rational functions with poles on $\overline{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$, which obtains a formulation allowing for multiple poles and a proof of invariance with respect to $\overline{\mathbb{R}}$-preserving Möbius transformations. In this work, we obtain a characterization of Stahl–Totik regularity of a GMP matrix in terms of its matrix elements; as an application, we give a proof of a conjecture of Simon – a Cesàro–Nevai property of regular Jacobi matrices on finite gap sets. This is joint work with Benjamin Eichinger and Milivoje Lukić.

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